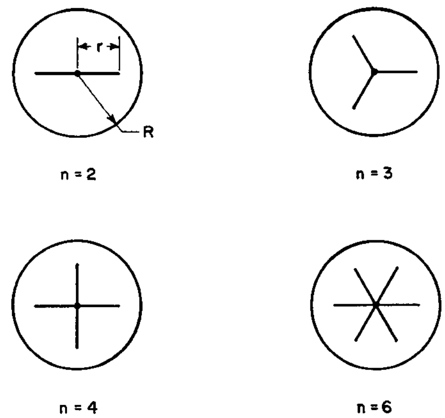

Fig. 1. Cross sections of n-fin transmission lines.

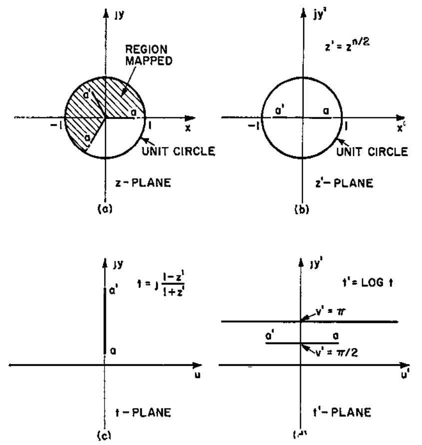

Fig. 2. Conformal mapping transformations. (a) Initial n-fin line ($n=3$ illustrated). (b) Two-fin line. (c) Intermediate transformation. (d) Strip transmission line.

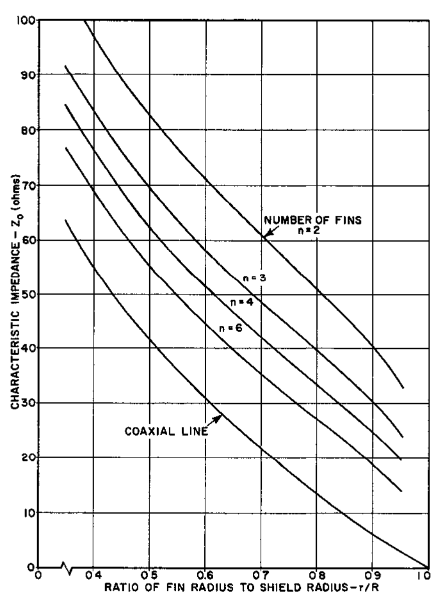

Fig. 3. Characteristic impedance of multifin lines.

The next step is to apply the transformation

$$t = j\frac{1-z'}{1+z'} \tag{4}$$

to the two-fin configuration. This transformation maps the unit circle of the z' plane onto the real axis of the t plane, and maps the fin on the real axis of the z' plane onto a portion of the imaginary axis of the t plane, as shown in Fig. 2(c). Finally, the t plane geometry is converted into a symmetric strip transmission line by the transformation

$$t' = \log t. \tag{5}$$

This relationship maps the positive real axis of the t plane onto the real axis of the t' plane, the negative real axis of the t plane onto the real axis of the t' plane, the negative real axis of the t plane onto the line $t'=u'+j\pi$, and the positive imaginary axis of the t plane onto the line $t'=u'+j\pi/2$. The portion of the t plane imaginary axis identified with the transformed fin turns out to be symmetrically positioned in the fashion of a strip transmission line. Consequently, the characteristic impedances of the strip transmission line and two-fin line will be equal when

$$\frac{w}{b} = \frac{2}{\pi}\log\left\{\frac{1+\frac{r_2}{R_2}}{1-\frac{r_2}{R_2}}\right\} \tag{6}$$

where w is the width of the strip conductor and b the ground-plane spacing.

Equations (2), (3), and (6) have been used along with data[1] on the characteristic impedance of zero-thickness strip transmission line to generate the curve of Fig. 3 for zero-thickness multifin lines. These curves show the dependence of characteristic impedance on the ratio r/R for lines of two, three, four, and six fins. Since it might be expected intuitively that a line with a large number of fins would begin to approach the r/R values of an ordinary coaxial line, this latter case has been plotted for comparison.

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[1] S. B. Cohn, "Problems in strip transmission lines," *IRE Trans. Microwave Theory and Techniques*, vol. MTT-3, pp. 119–126, March 1955.

Comment on "Cylindrical Waveguides Containing Inhomogeneous Dielectric"

In the above correspondence, AhSam and Klinger[1] discussed the difficulty of obtaining complete analytic solutions for propagation of electromagnetic waves in inhomogeneous media with circular cylindrical symmetry. In order to aid the search for exact solutions, they studied the special case in which the relative permittivity, independent of the azimuthal and axial coordinates, varies inversely as the square of the radius so that two of the six field equations uncouple.

I should like to point out that AhSam and Klinger overlooked the existence of certain classes of modes. That modes for nonzero azimuthal variation (i.e., $n\neq 0$ in their notation) *must* exist, independently of the relationship among a, b, and l (the radii of the metal cylinders containing the inhomogeneous medium and the constant entering the relative permittivity), can be deduced from the following arguments. First, as the frequency is raised, the medium appears more and more uniform locally, and a high-frequency wave launched in some direction not in the r-z plane should have no difficulty propagating down the waveguide. Secondly, since the modes in any closed cylindrical waveguide form a complete set, one should be able to expand a function of θ in terms of it, so that the set must contain modes with $n\neq 0$.

Except in very special cases, azimuthally dependent modes in circular cylindrical structures are hybrid modes, since it is generally not possible to satisfy all the boundary conditions with just TE or TM. Thus, AhSam and Klinger are correct in stating that TM solutions (only f_2, g_1, and g_3 nonzero) and TE solutions (only g_2, f_1, and f_3) do not exist for $n\neq 0$. It is also true that all six components cannot exist simultaneously (except for the special relations between a, b, and l) since the two characteristic equations (13) and (18)[1] would not have simultaneous solutions. However, it is possible to have modes with nonzero f_1, f_2, f_3, g_1, and g_3, or f_1, f_3, g_1, g_2, and g_3 which satisfy all the given conditions. In the first case, f_2 is given as in (12),[1] with

$$f_1 = \frac{\beta}{\alpha k_z}\frac{d}{dr}(rf_2)$$

$$f_3 = -\frac{j}{\alpha}r\beta f_2$$

$$g_1 = \frac{-k_z}{(\omega\mu)^2}\frac{k_0{}^2 l}{\alpha}f_2$$

$$g_3 = j\frac{k_0{}^2 l}{\alpha(\omega\mu)^2}\frac{1}{r}\frac{d}{dr}(rf_2)$$

and k_z determined by (13). Similarly, a possible solution using (18) to determine k_z has g_2 as given by (16)[1] and

$$f_1 = \frac{r^2 k_z}{\alpha}g_2$$

$${}_3 = -\frac{j}{\alpha}\left[r^2\frac{dg_2}{dr} + rg_2\right]$$

$$g_1 = -\frac{\beta}{\alpha k_z}\frac{d}{dr}(rg_2)$$

$$g_3 = jr\frac{\beta}{\alpha}g_2.$$

Manuscript received March 23, 1967.

[1] E. AhSam and Y. Klinger, *IEEE Trans. Microwave Theory and Techniques (Correspondence)*, vol. MTT-15, p. 60, January 1967.

Both of these classes of modes are hybrid, since neither E_z nor H_z (f_3 or g_3) is zero. However, the modes are TM and TE, respectively, with respect to the θ coordinate, a result due to the inverse square dependence of ϵ_r which uncoupled (5) and (8).[1] For the case that $n=0$, so that $\beta=0$, the above modes reduce to the TE and TM waves found by AhSam and Klinger.

It should also be pointed out that the "cutoff condition" of (14)[1] may more correctly be called a "killoff" condition, as discussed by Zucker.[2] The modes cease to exist altogether when $k_0{}^2l$ becomes less than $1+n^2$. Cutoff occurs at the frequency for which $k_z=0$, below which the wave might still exist although evanescent. The cutoff conditions can be determined by solving (13) and (18).

Lastly, it may be noted that there *have* been previous "analytic solutions for both TE and TM type modes in inhomogeneous media in cylindrical coordinates." As with AhSam and Klinger's example, the previous examples were also for a special case of permittivity variation, but included full discussion of the complete sets of hybrid modes. Representative references for these examples of propagation in cylinders with inhomogeneous media are for closed waveguides[3] and for open structures.[4]

[2] F. J. Zucker, "Electromagnetic boundary waves—an introduction," Air Force Cambridge Research Laboratories, Bedford, Mass., Rept., June 1963.

[3] P. J. B. Clarricoats, "Propagation along unbounded and bounded dielectric rods, pt. 2," *Proc. IEE* (London), vol. 108C, p. 177, October 1960.

[4] S. P. Schlesinger, P. Diament, and A. Vigants, "On higher-order modes of dielectric cylinders," *IRE Trans. Microwave Theory and Techniques*, (*Correspondence*), vol. MTT-8, pp. 252–253, March 1960.

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Authors' Reply[5]

We are indebted to Richter for pointing out these modes which we overlooked. The cutoff conditions equation (14) or (18)[1] apply to these modes too.

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[5] Manuscript received May 10, 1967.

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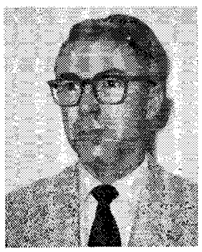

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